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A Hybrid Discontinuous Galerkin Method with Impedance Traces for the Helmholtz Equation

The Helmholtz equation with absorbing boundary conditions has been the focus of much research. It describes a time-harmonic wave propagation. Due to the oscillatory behaviour of solutions for the Helmholtz equation, developing simulation methods is challenging. If the finite element method is applied, then a fine mesh size is required leading for many wavelengths in the computational domain to large systems of linear equations, which need to be solved. Direct solver can be applied to solve these problems, but the memory consumption of such solvers is too demanding and increases too quickly with respect to the mesh size. Preconditioned iterative solvers are an answer to these issues, and for coercive (elliptic) problems, such have been developed and applied to great success, e.g. multi-grid. Local smoothing steps, in combination with direct coarse grid corrections, lead to very satisfactory results. Sadly, the Helmholtz equation is not coercive and exhibits a non-local behaviour; these iterative solvers can not be applied. The idea of using domain decomposition preconditioners with some minimal residual iterative solver has been devised. The Helmholtz equation needs to be solved on subdomains and therefore preconditioners based upon domain decomposition need to be absolutely stable in the sense that the local preconditioning problems are always uniquely solvable. It has been shown that discontinuous Galerkin methods exhibit a local stability property, leading to stably solvable problems on subdomains. By introducing discontinuous Galerkin methods, the already large system of linear equations is substantially increased due to the duplication of coupling unknowns. The hybrid discontinuous Galerkin methods with static condensation capabilities have been developed to counteract this issue. All volume unknowns are condensed to skeleton unknowns, leading to a system of linear equations only for these skeleton unknowns. The number of skeleton unknowns is for small polynomial degrees larger than the number of unknowns for conforming spaces, but hybrid discontinuous Galerkin methods exhibit less unknowns as discontinuous Galerkin methods. The property of being able to apply static condensation is highly non-trivial for the Helmholtz equation, but can be directly derived from a local stability property of hybrid discontinuous Galerkin methods. Not all hybrid discontinuous Galerkin methods are suitable for domain decomposition preconditions in the context of the Helmholtz equation. The transmission conditions between subdomains are crucial such that they represent impedance traces. The focus of this dissertation is exactly on a hybrid discontinuous Galerkin method exhibiting all of these favourable properties. It is locally absolutely stable, exhibits optimal convergence rates with respect to the mesh size, and iterative solvers with preconditioners based on domain decomposition concepts can be applied. In this work, the rigorous stability and error analysis of the hybrid discontinuous Galerkin method is carried out, which is a novelty. Additionally, the favourable properties concerning iterative solvers of the method are highlighted by large-scale numerical simulations.