Sparse Channel Estimation for Vehicular Millimeter Wave Communication

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Why Directive Antennas for Vehicular mmWave

- Scaling in frequency decreases free space path loss as the aperture decreases
- Keeping aperture (power) constant, increases gain of antennas, thus beams are getting narrower
- Narrow beam have the down-side of being susceptible to alignment errors.
- Mmwave transceivers ideally as fully digital architecture with independent RF chains
  $\rightarrow$allows MIMO
- Cost and power consumption constrain transceivers architecture to more hybrid approaches, e.g. with array of sub-arrays
- allows beam steering on sub-array level
  $\rightarrow$a beam per sub-array
- Dynamic control and management of links is a major challenge for mmWave V2X
  $\rightarrow$time-variant channel

OFDM channel estimation

- OFDM wireless communication systems
- Coherent detection require accurate channel state information (CSI)
- Channel estimation based on pilot symbols $p[k, l]$ or data symbols
- pilots scattered in time and frequency for doubly-spread channel
- e.g. estimate with MMSE or zero-forcing from received symbols $r[k, l]$

$$\hat{H}[k, l] = \frac{r[k, l]}{p[k, l]} = H[k, l] + z[k, l]$$

Doubly-Spread Channels

- Scattering function of a WSS process describes dispersion of the channel in delay and Doppler frequency

$$|s(\nu,\tau)|^2 = \left|\int_{-\infty}^{\infty} h(t,\tau)\exp(-j2\pi\nu t)\,dt\right|^2$$

- Vehicular channel not WSS[1]
- Evaluate in local scales T_{eval} where WSS assumption holds
- Multitaper estimator of scattering function known as local scattering function

$$|s(q\cdot T_{\text{eval}};\nu,\tau)|^2 \quad \begin{array}{ll} q & \ldots\text{evaluation window} \\ \nu & \ldots\text{Doppler frequency} \\ \tau & \ldots\text{delay} \end{array}$$

[1] Matz: On non-WSSUS wireless fading channels, 2005

Beamforming Implications on Wireless Channels

- Jakes'/Clarke's Doppler power spectrum for uniform distributed scatterer
$\rightarrow$U-shaped
- Beamforming reduces the Doppler spread due to spatial filtering
- Narrow beams decrease small-scale fading depth as number of MPCs is decreased
- Delay spread is reduced
- Coherence bandwidth increases with an increase in antenna directivity[2]
- Narrow beam limits channel coherence time[3]

[2] N. Iqbal et al.: Investigating Validity of WSS Assumption in mmWave Radio Channels
[3] V. Va et al.: Impact of beamwidth on temporal channel variation in veh. channels and its implications

Sparse Channel Structure

- Simple multipath channel model

$$h(t,\tau) = \sum_{p=1}^{P} \eta_p \delta(\tau - \tau_p) e^{j2\pi\nu_p t} d\tau d\nu$$

- Resolve multipath arrivals in delay with sufficiently wide bandwidth
  $\rightarrow$large available bandwidths at mmWave
- Slowly time-variant channel
  $\rightarrow$Delays approx. constant during one OFDM symbol (q)
- $H(t,f)^{(q)} = \sum_p^P \eta_p e^{-j2\pi\tau_p^{(q)} f} e^{j2\pi\nu_p^{(q)} t} + z(t,f)^{(q)} \quad t^{(q-1)} < t \leq t^{(q)}$
- Delays τ_p and Doppler shifts ν_p valuable for beam-steering and channel tracking

Atomic Norm for 2D

- 2D-signal $\mathsf{Y} = \mathsf{X} + \mathsf{N} = \sum_p^P \eta_p \mathsf{a}_\nu(\nu_p)\mathsf{a}_\tau(\tau_p)^\mathrm{T} + \mathsf{N}$
- Atomic set $\mathcal{A}$

$$\mathcal{A} = \{e^{j\phi}\mathsf{a}_\nu(\nu)\mathsf{a}_\tau(\tau)^\mathrm{T} \,:\, \nu \in [0,1), \tau \in [0,1), \phi \in [0, 2\pi)\},$$

$$\mathsf{a}_\tau(\tau_p) = [1 \; e^{j2\pi\tau_p} \; \dots \; e^{j2\pi(N_\tau - 1)\tau_p}]^\mathrm{T},$$

$$\mathsf{a}_\nu(\nu_p) = [1 \; e^{j2\pi\nu_p} \; \dots \; e^{j2\pi(N_\nu - 1)\nu_p}]^\mathrm{T},$$

- Atomic norm $\|\mathsf{x}\|_\mathcal{A}$ where $\mathsf{x} = \mathrm{vec}(\mathsf{X})$

$$\|\mathsf{x}\|_\mathcal{A} = \inf\{t \geq 0 | \mathsf{X} \in t \cdot \mathrm{conv}(\mathcal{A})\} = \inf\left\{\sum_p |\eta_p| \middle| \mathsf{X} = \sum_p \eta_p \mathsf{a}_\nu(\nu_p)\mathsf{a}_\tau(\tau_p)^\mathrm{T}\right\}$$

- Semidefinite program (SDP) to approximately calculate $\|\mathsf{x}\|_\mathcal{A}$ [4]
  $\rightarrow$atomic decomposition

[4] Y. Chi et al.:Compressive two-dimensional harmonic retrieval via atomic norm minimization

Atomic Norm Denoising

- SDP in presence of noise N

$$\begin{aligned} &\underset{\mathsf{X},\mathsf{u},t}{\text{minimize}} && \frac{\mu}{2N}\operatorname{tr}\left(\mathsf{T}_2(\mathsf{u})\right) + \frac{\mu}{2}t + \frac{1}{2}\|\mathsf{Y}-\mathsf{X}\|_F \\ &\text{subject to} && \begin{bmatrix} \mathsf{T}_2(\mathsf{u}) & \mathsf{x} \\ \mathsf{x}^H & t \end{bmatrix} \succeq 0, \end{aligned} \tag{1}$$

- Toeplitz block-Toeplitz matrix $\mathsf{T}_2(\mathsf{u})$ fully parameterized by u
- Regularizer μ tunes trade-off between sparsity and quality of reconstruction $\|\mathsf{Y}-\mathsf{X}\|_F$
- Frequencies (ν_p, τ_p) often retrieved from dual polynomial of SDP
- Alternatively, retrieve (ν_p, τ_p) via Vandermonde decomposition of $\mathsf{T}_2(\mathsf{u})$

Sparse channel estimate

- For $\mathrm{rank}(\mathsf{T}_2(\mathsf{u})) \leq \min(N_\nu, N_\tau)$, Vandermonde decomposition unique

$$\mathsf{T}_2(\mathsf{u}) = V\mathsf{D}V^{\mathrm{H}} = \sum_{p}^{P} |\eta_p^{(q)}|^2 [\mathsf{a}_\tau(\tau^{(q)})\mathsf{a}_\tau(\tau^{(q)})^{\mathrm{H}}] \otimes [\mathsf{a}_\nu(\nu^{(q)})\mathsf{a}_\nu(\nu^{(q)})^{\mathrm{H}}]$$

- Matrix pencil and pairing (MaPP) method to retrieve $(\nu_p^{(q)}, \tau_p^{(q)})$ from $\mathsf{T}_2(\mathsf{u})$
- Construct basis matrix $\mathsf{B} = \left[\mathsf{a}_\tau(\tau_1^{(q)}) \otimes \mathsf{a}_\nu(\nu_1^{(q)}) \quad \dots \quad \mathsf{a}_\tau(\tau_P^{(q)}) \otimes \mathsf{a}_\nu(\nu_P^{(q)})\right]$
- Complex path gain from least-squares solution

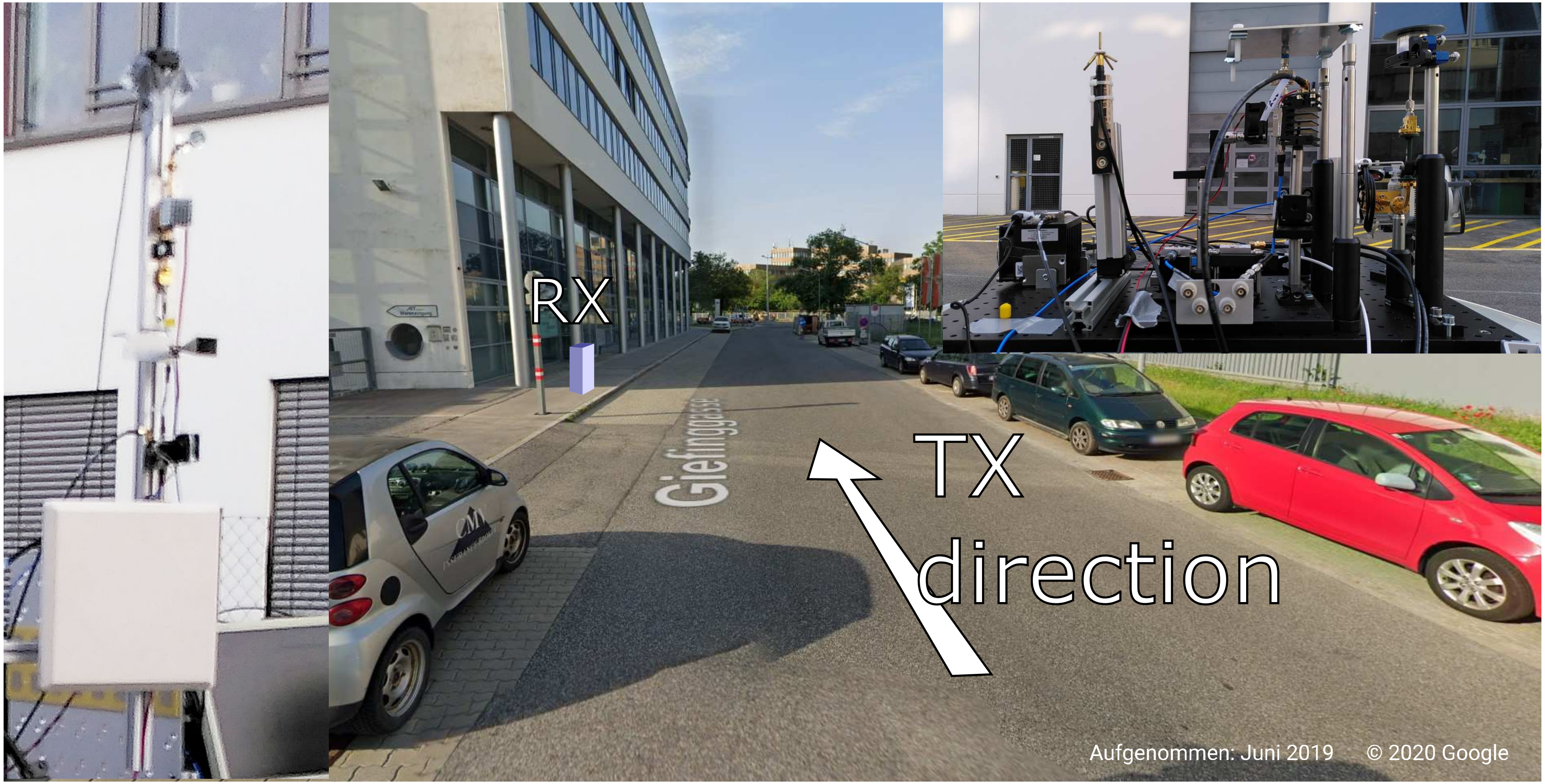
$$\eta^{(q)} = \mathsf{B}^\dagger \mathrm{vec}(\mathsf{H}^{(q)})$$

- Sparse channel representation

$$\mathsf{H}^{(q)} = \sum_{p}^{P} \eta_p^{(q)} \mathsf{a}_\nu(\nu_p^{(q)}) \mathsf{a}_\tau(\tau_p^{(q)})^{\mathrm{T}}$$

AIT Tripleband Measurement Campaign

- synchronized vehicular measurement at three bands[5], $f_c = 3.2, 34.3, 62.35\,\mathrm{GHz}$
- moving TX omni-directional, static RX with 20° beamwidth

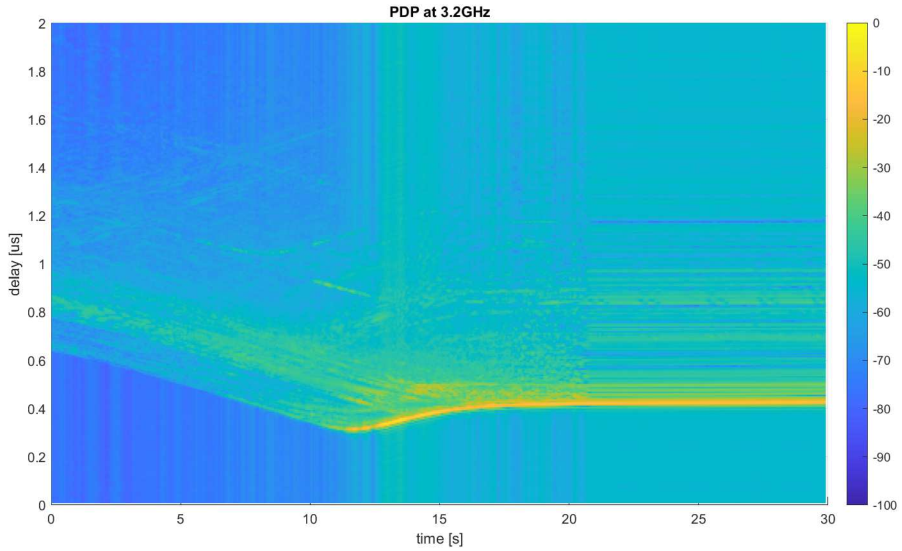


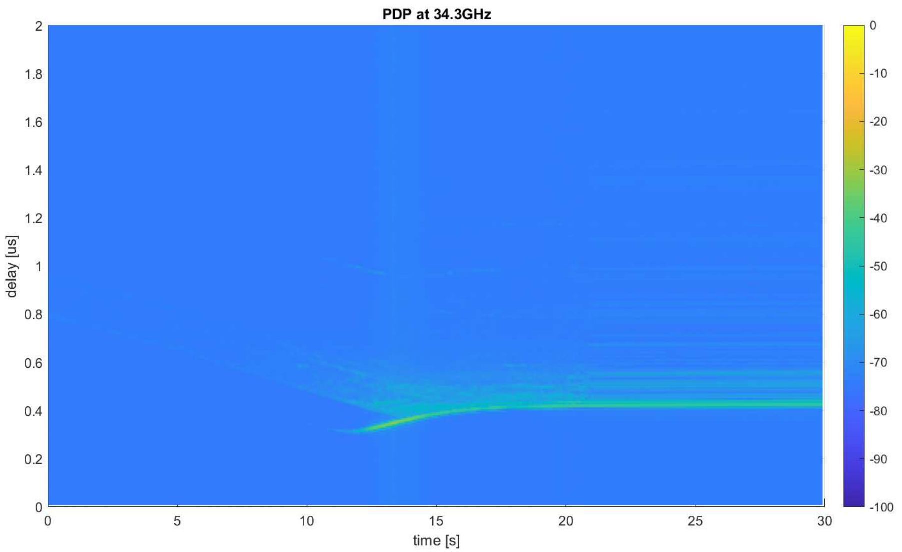
[5] M. Hofer et al.: PIMRC, 2021

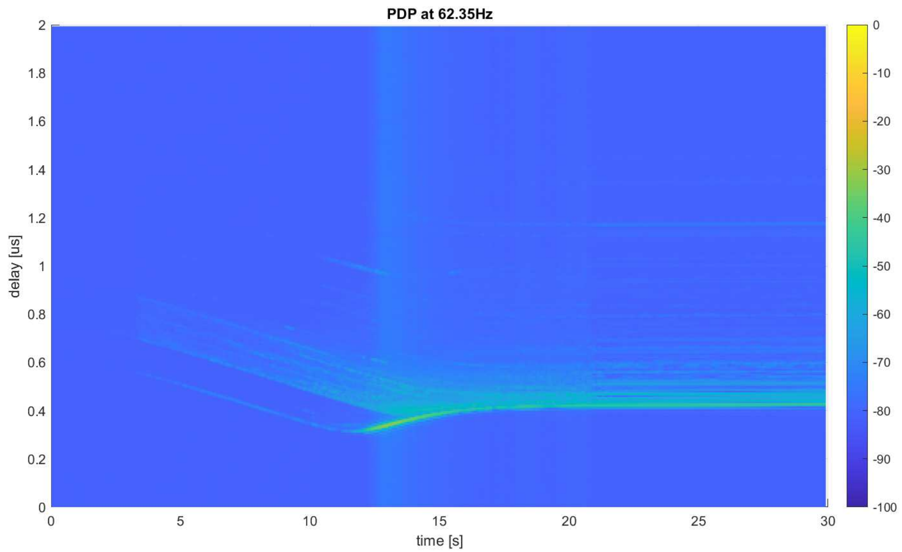
AIT Tripleband Measurement Campaign: Power Delay Profiles

- Power delay profile (PDP)

$$P_h(q \cdot T_{\text{eval}}; \tau) = \int \left| s\left(q \cdot T_{\text{eval}}; \nu, \tau\right) \right|^2 d\nu$$





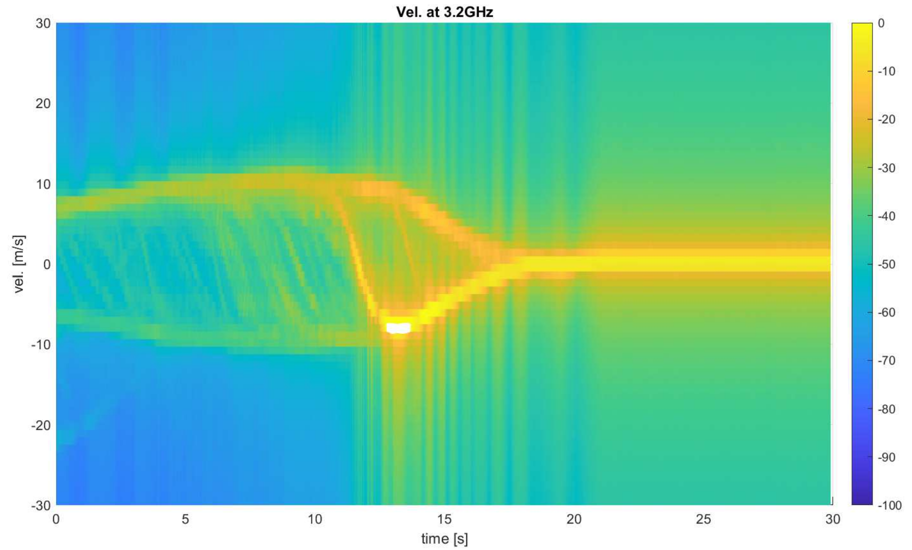


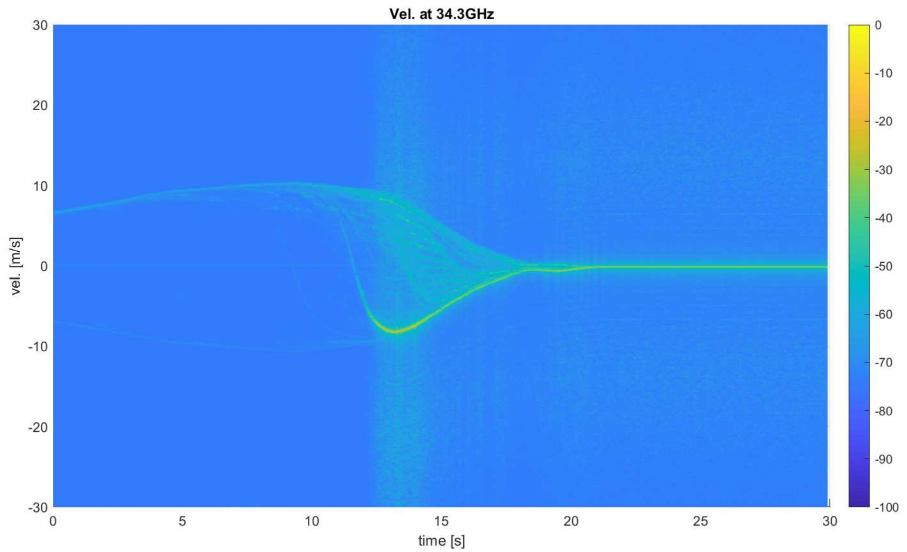
AIT Tripleband Measurement Campaign: Power Doppler Profiles

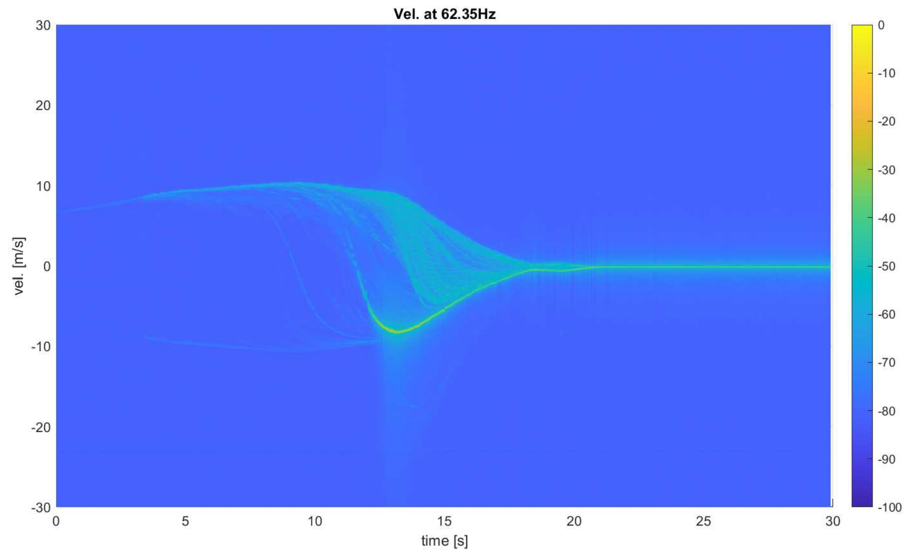
- Power Doppler profile

$$P_B(q \cdot T_{\text{eval}}; \nu) = \int \left| s\left(q \cdot T_{\text{eval}}; \nu, \tau\right) \right|^2 d\tau$$

- normalized to velocity for comparison

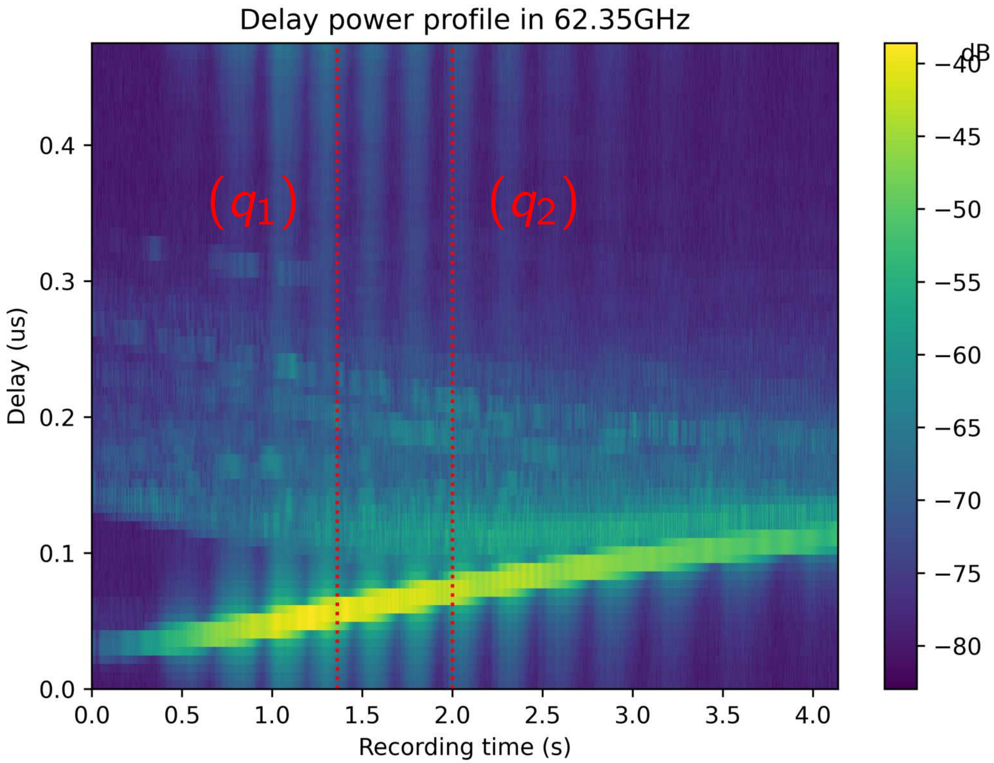


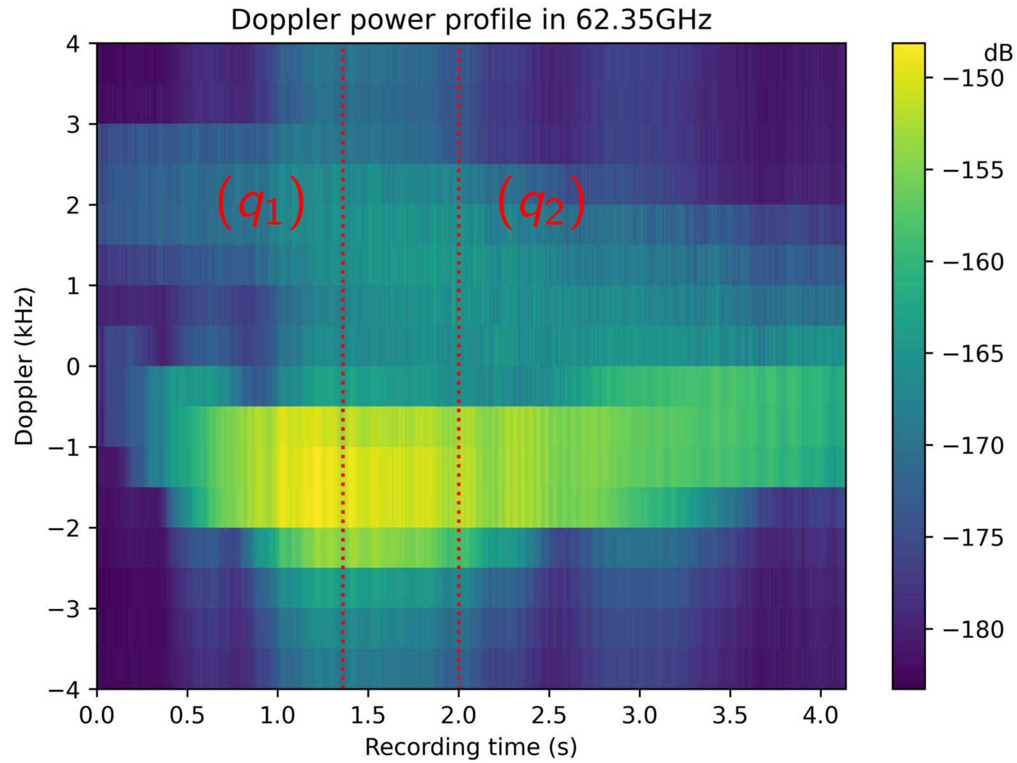




Sparse Channel Estimation at Millimeter Wave Band

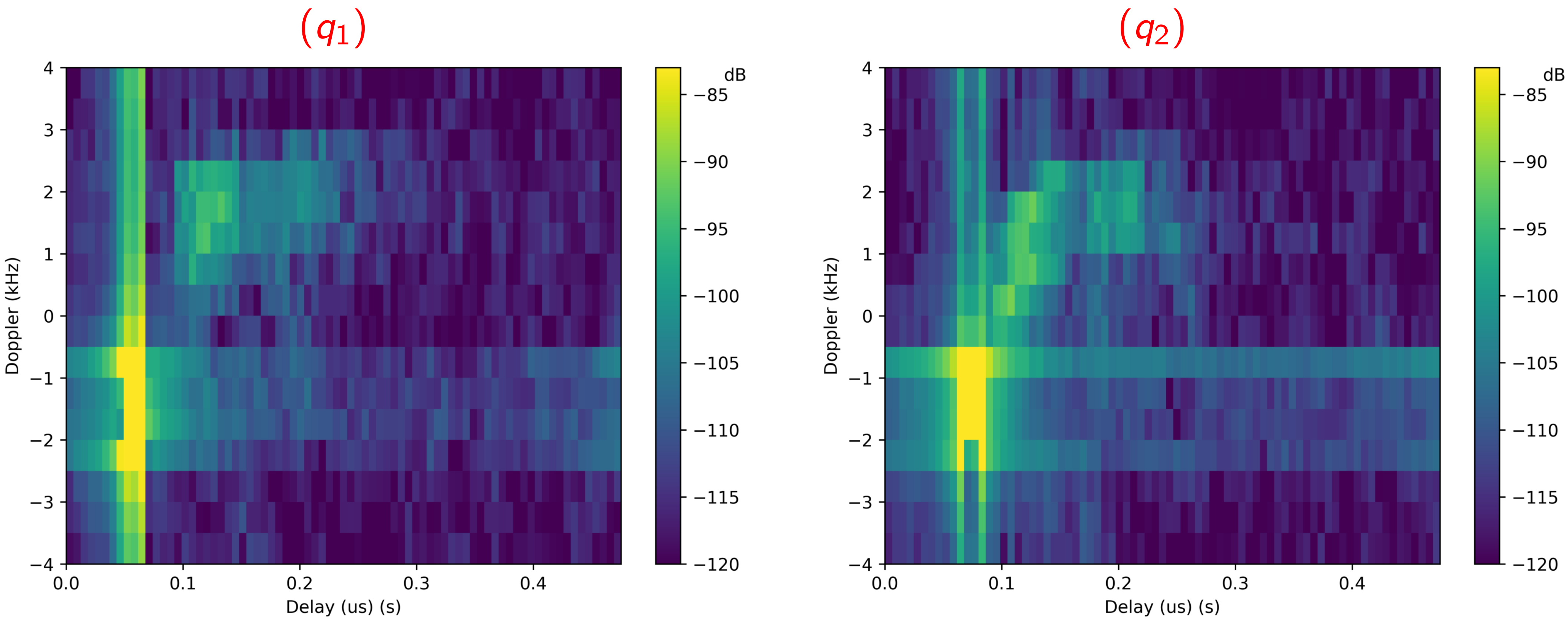
- 77 subcarriers
- subcarrier spacing 2 MHz
- symbol rate 125 μs
- 16 symbols, i.e. $T_{\text{eval}} = 2\,\text{ms}$



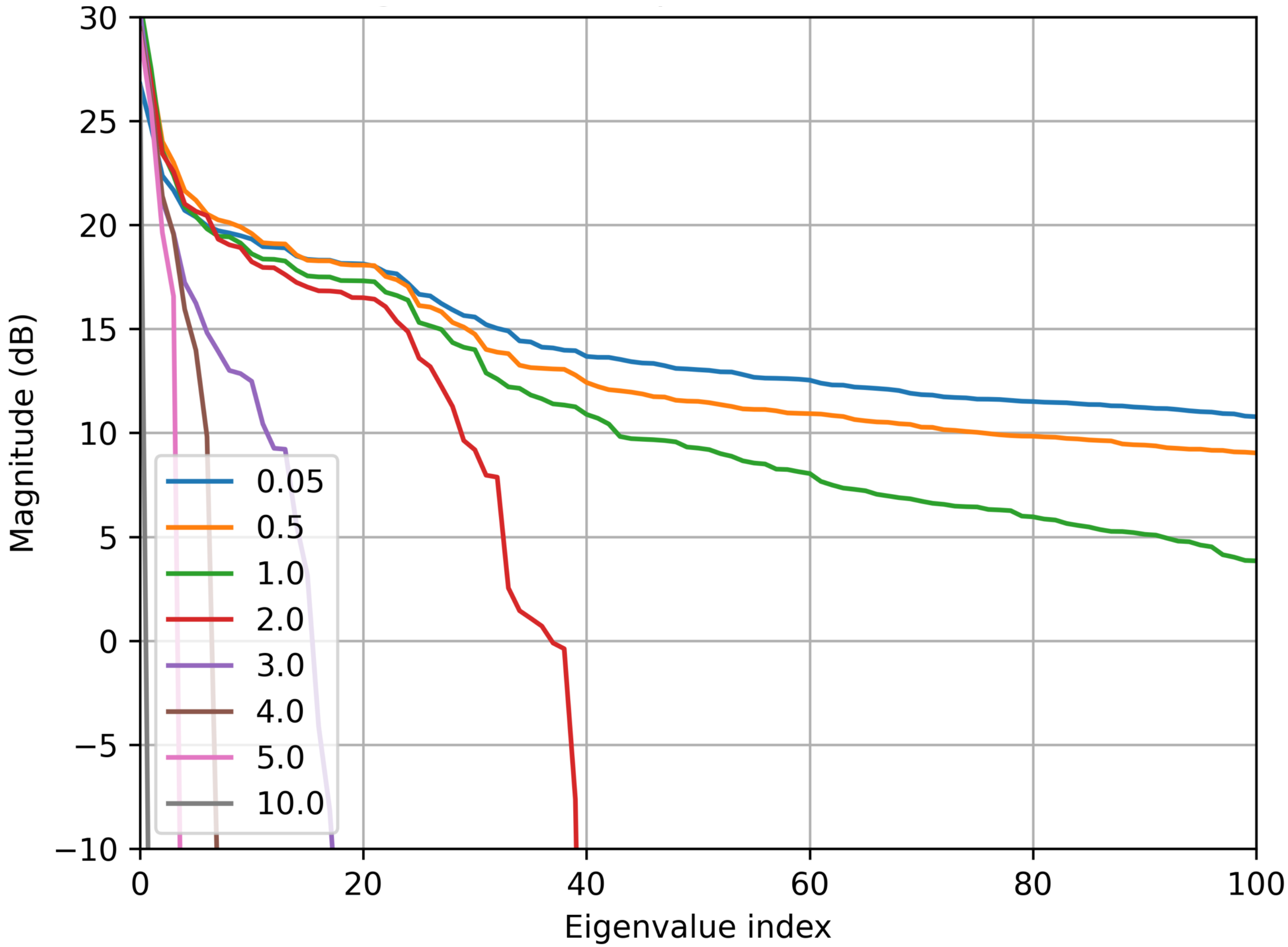


Delay-Doppler Scattering Function Limitations

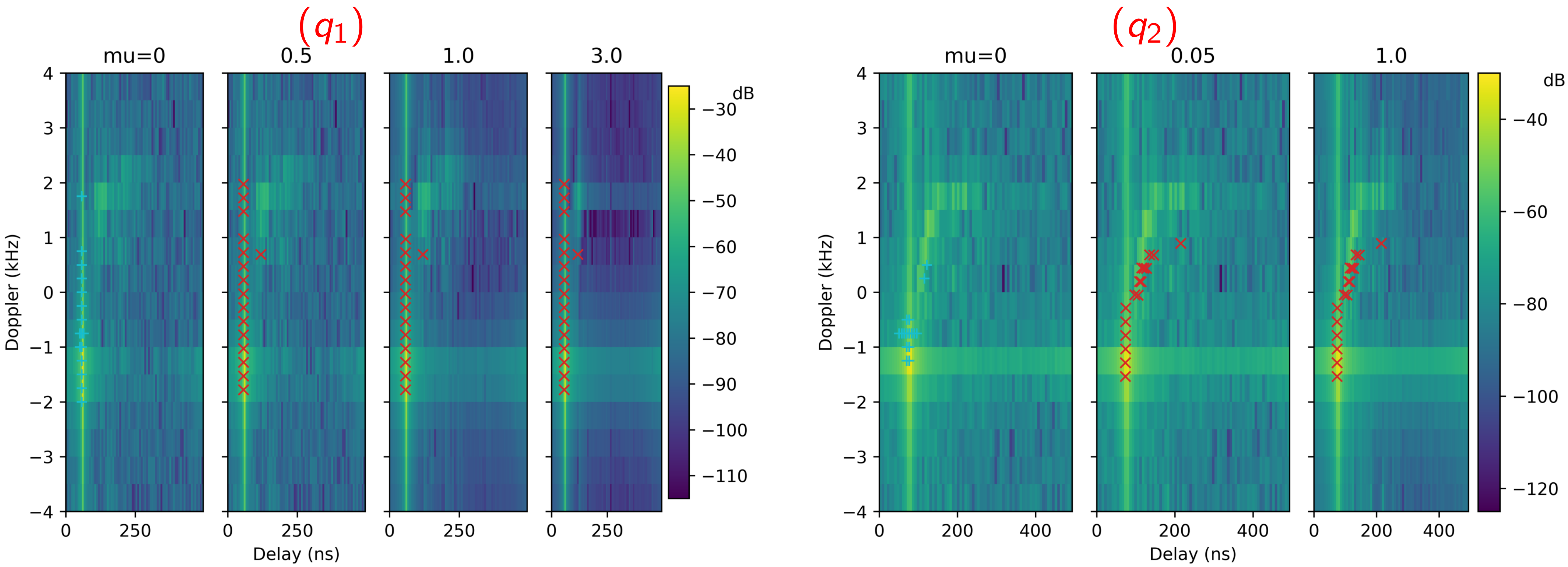
- Spreading function $s_{\mathrm{H}}(\tau, \nu)$ only approximately sparse
- reflections by extended objects
- delay/Doppler leakage due to band or time limitations at transmitter/receiver



Eigenvalues of Toeplitz Matrix from Atomic Norm Denoising



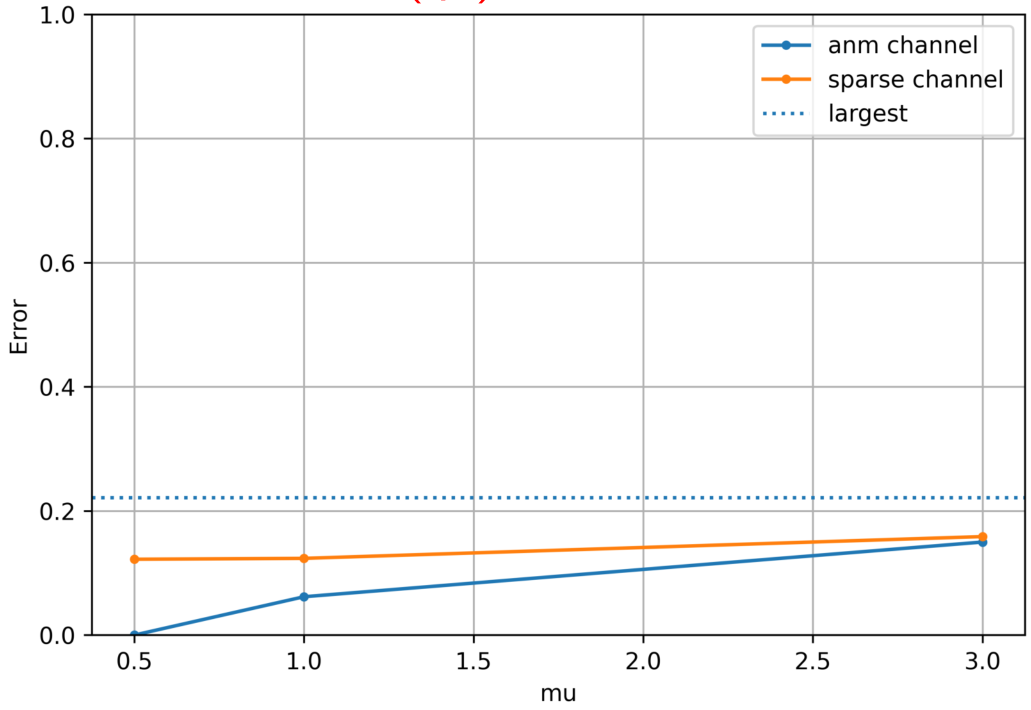
Sparse spreading function



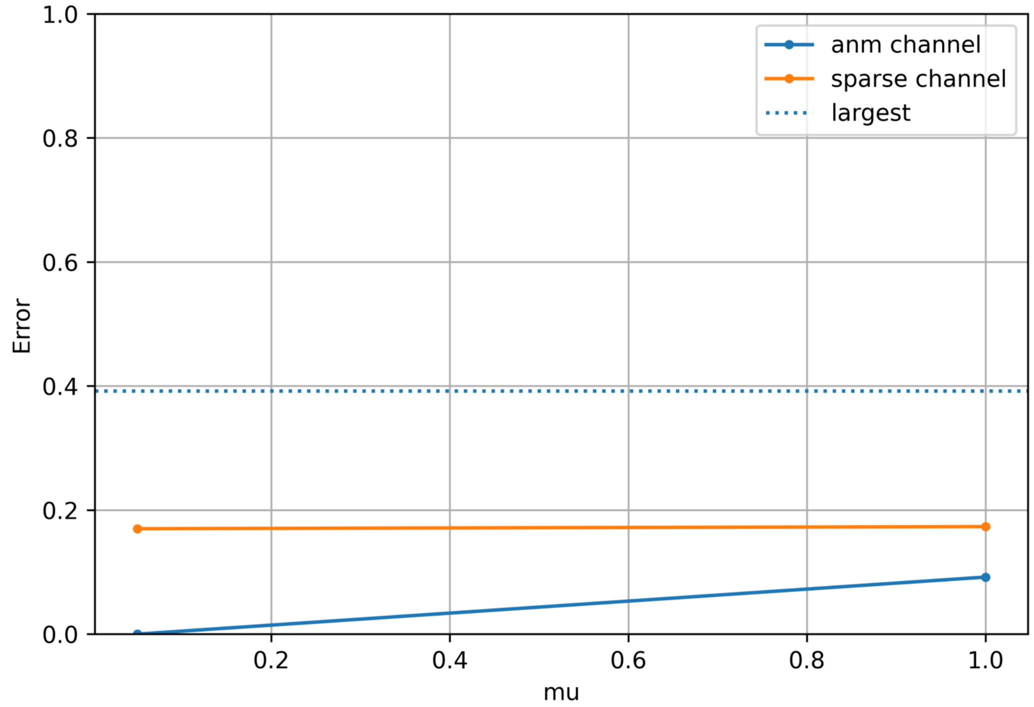
Error of reconstruction

- ANM channel error $\|\mathrm{H}_{\mathrm{ZF}} - \mathrm{X}\|_F / \|\mathrm{H}_{\mathrm{ZF}}\|_F$
- sparse ANM channel error $\|\mathrm{H}_{\mathrm{ZF}} - \mathrm{H}^{(q)}\|_F / \|\mathrm{H}_{\mathrm{ZF}}\|_F$

(q_1)



(q_2)



Conclusion

- Application of atomic norm to vehicular millimeter-wave measurements
- Atomic norm denoising framework allows retrieval of sparse representation of slowly-varying channel in delay-Doppler
- Deal with decay of clusters
- Atomic norm minimization (ANM) still infeasible for large dimensions
- Decoupled 2D ANM reduces complexity[6]
  $\rightarrow$Delay and Doppler estimated separately, but needs coupling

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- Thank you!

[6] H.Groll et al.: Sparse Approximation of an Outdoor-to-Indoor Massive MIMO Channel Measurement