Time-Varying Packet Loss Models for Networked Control

SPSC, TUGraz

September 7, 2021

Hadi Gharibdoust, Gernot Kubin

Table of Contents

1. Problem Statement

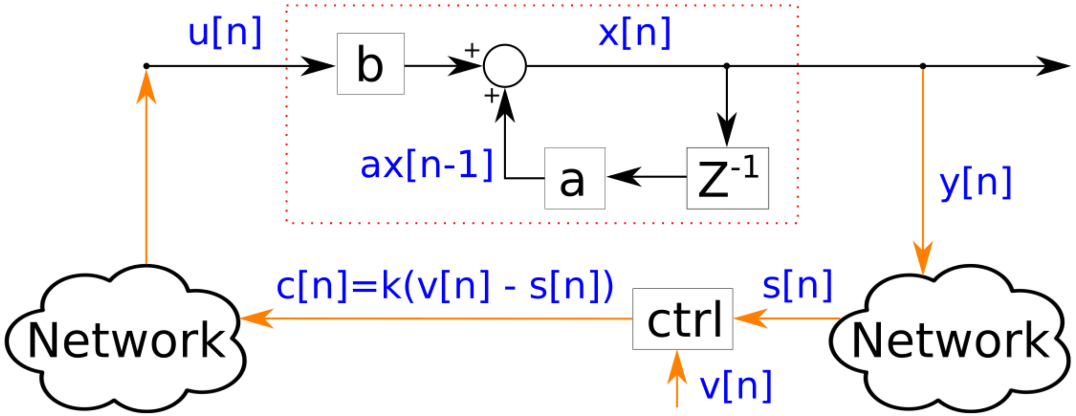
Packet Loss in
Networked Control Systems

Example: Feedback Control of an Unstable Plant (1)

Tracking system with a sinwave refernce input:

General description:

- Unstable open-loop plant (|a| >= 1).
- Feedback used for stabilization (a-bk < 1).



Question:

In the linear state feedback mechanism
what happens if:

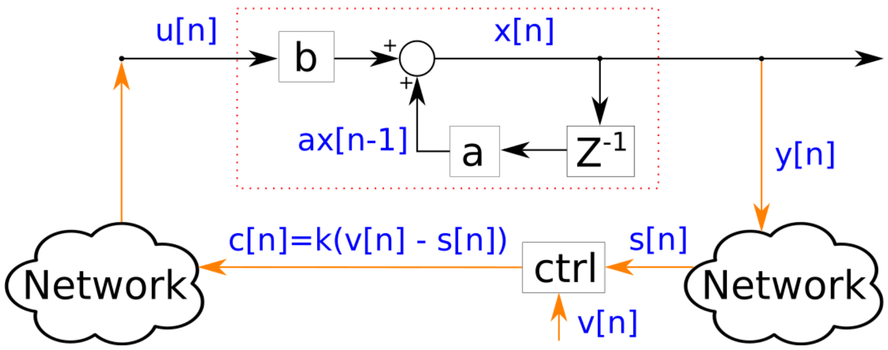
Only every 16th feedback control sample c[n] reachs u[n] (zero-order hold):

$$u[n]=c\left[\ 1+16\left\lfloor\frac{n-1}{16}\right\rfloor\ \right]$$

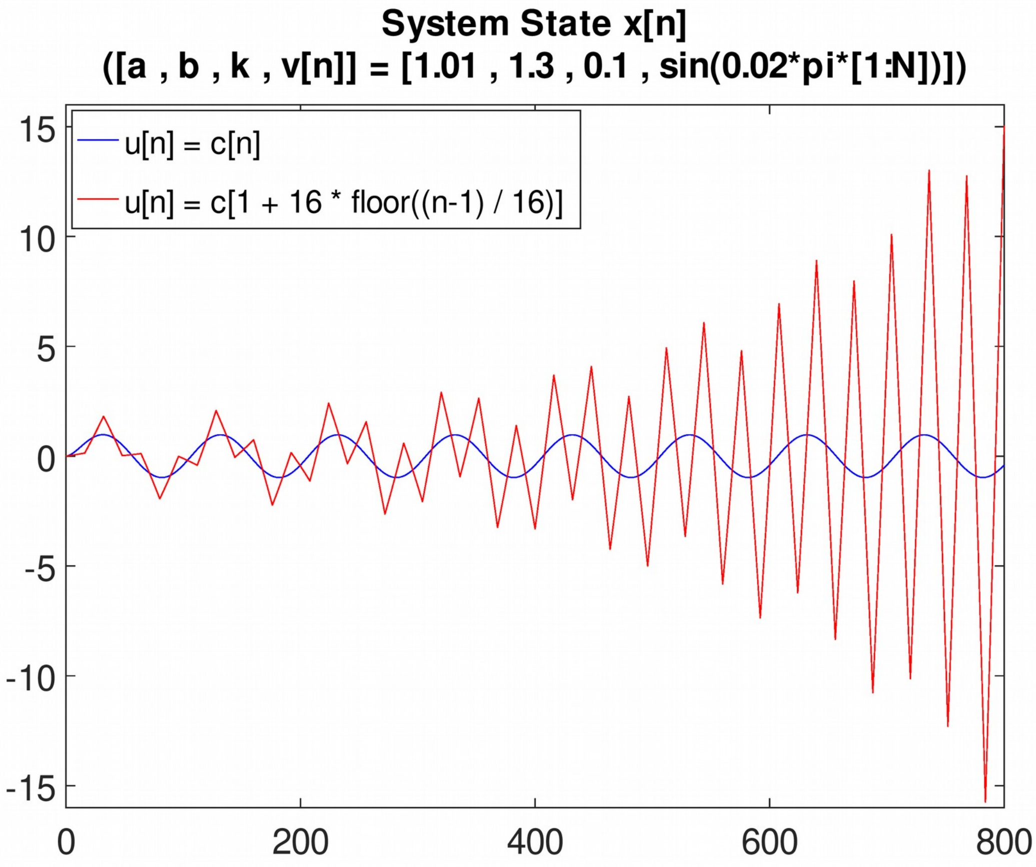
Example 1: Feedback Control of an Unstable Plant (2)

Numeric example

- Unstable open-loop plant (a >= 1).
- Feedback used for stabilization.
- Stable closed-loop (a-bk < 1).



Question:
What happens if:
The decimation factors at any moment is a stochastic random variables?



2. Reality Check: ITI@TUGraz Testbed

2-1. Interference Source

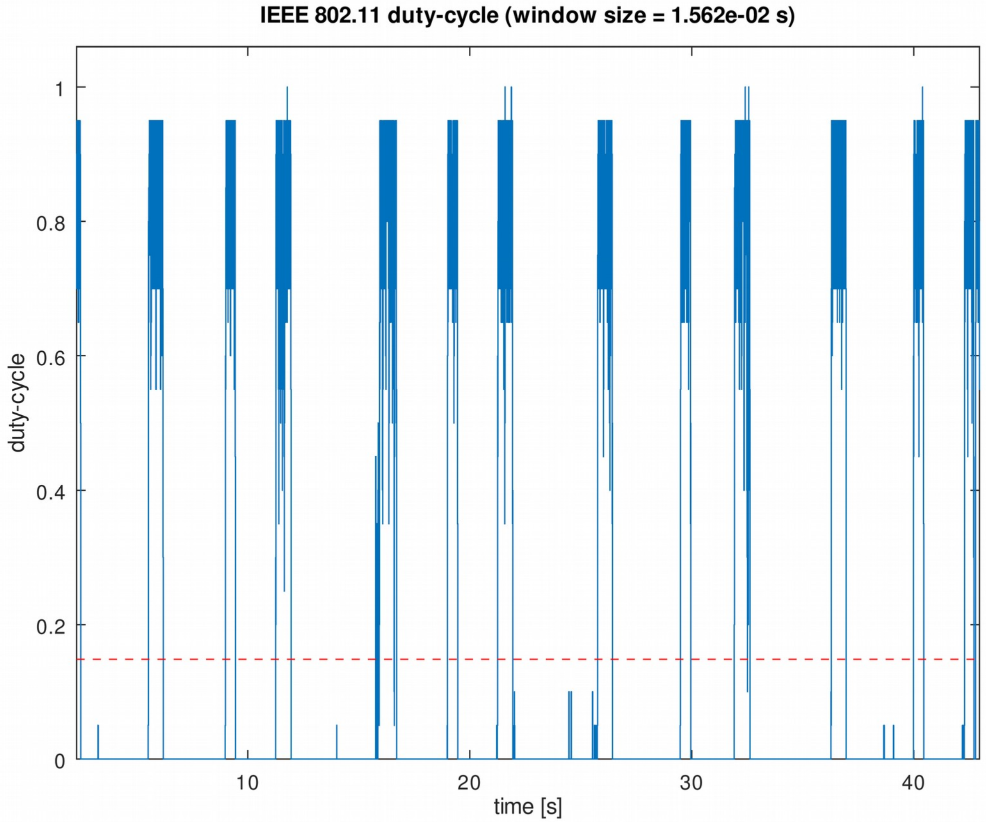
Collision Probability Estimation

Example: A sample IEEE 802.11 radio traffic (application: youtube stream):

- Traffic pattern

  1) Application layer protocol impact: slow & considerable

  2) MAC layer protocol impact too fast for adaptation & relatively less significant.



2. Reality Check: ITI@TUGraz Testbed

2-2. Communication Link

Estimation procedure:

1) estimating sample PMF using histogram

1-1) Observation PMF if process is i.i.d. (thus stationary & independent)

1-2) Higher order joint observation PMF if process is stationary

1-3) Conditional observation PMF if the process is only homogeneous

2) smoothing the PMF if needed

3) Corresponding population PMF = expected sample PMF

4) determine PMF estimation error

Example: IEEE 802.15.4 packet loss under the IEEE 802.11 Interference:

```
observation pmf     : [received , lost] = [ 0.876 , 0.124 ]

2nd order joint pmf : [received-to-received , received-to-lost    = [ 0.777 , 0.099
                       lost-to-received     , lost-to-lost     ]     0.099 , 0.025 ]

conditional pmf    : [if-received-to-received , if-received-to-lost  = [ 0.887 , 0.113
                       if-lost-to-received     , if-lost-to-lost     ]    0.798 , 0.202 ]
```

SPSC

Definition:

1) Random process X with random variables (RV) X_i and domain $\{x_1, x_2, ..., x_m\}$

2) Entropy $H(X_i)$:

$$H(X_i) = -E\left[\log p(X_i)\right] = -\sum_{j=1}^{m} p(X_i = x_j) \log p(X_i = x_j)$$

3) Conditional entropy of the adjacent RVs $H(X_{i+1} \mid X_i)$:

$$H(X_i \mid X_{i-1}) = -E\left[p(X_i \mid X_{i-1}) \log p(X_i \mid X_{i-1})\right] = -\sum_{j=1}^{m} p(X_i = x_j) \left(\sum_{k=1}^{m} p(X_i = x_k \mid X_{i-1} = x_j) \log p(X_i = x_k \mid X_{i-1} = x_j) \right)$$

4) Redundancy in X_i given X_{i-1} (dependency indicator):

$$H(X_i) - H(X_i \mid X_{i-1})$$

5) Relative redundancy:

$$\frac{H(X_i) - H(X_i \mid X_{i-1})}{H(X_i)}$$

Entropy Analysis

Example: IEEE 802.15.4 packet loss under the IEEE 802.11 Interference:

$p(X_i) = [\ 0.876\ ,\ 0.124\ ]$

$p(X_{i-1}\ ,\ X_i) = [\ 0.777\ ,\ 0.099\ ;\ 0.099\ ,\ 0.025\ ]$

$p(X_{i-1}) = p(X_{i-1}) = [\ 0.876\ ,\ 0.124\ ]$

$H(X_i\ |\ X_{i-1}) = H(X_{i-1}\ ,\ X_i) - H(X_{i-1})$

$H(X_i) = 0.54075$ [bit]

$H(X_{i-1}\ ,\ X_i) = 1.0765$ [bit]

$H(X_{i-1}) = 0.54075$ [bit]

$H(X_i\ |\ X_{i-1}) = 0.5357$ [bit/sample]

$H(X_i) - H(X_i\ |\ X_{i-1}) = 0.005$ [bit/sample]

$1 - H(X_i\ |\ X_{i-1})\ /\ H(X_i) < 1\%$

Conclusion:

1) 2 consecutive RVs : On average almost independent!

2) Packet loss probability: 1) Time varying from one moment to the next

2) Time variation rather periodically than stochastic.

(1,2) → Inhomogeneous Bernoulli Model

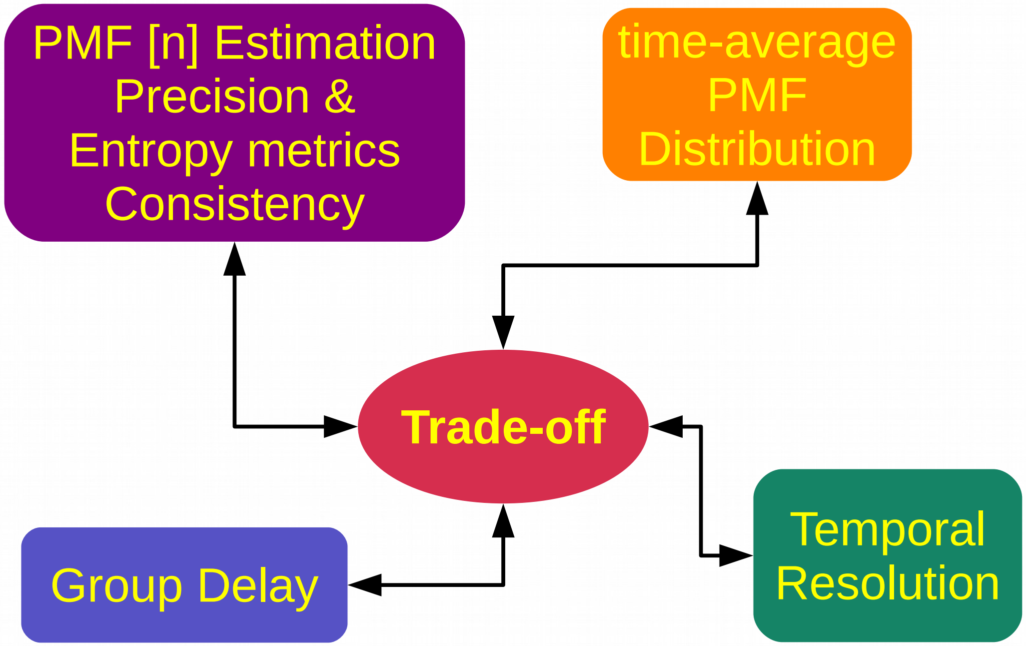
3. Proposal

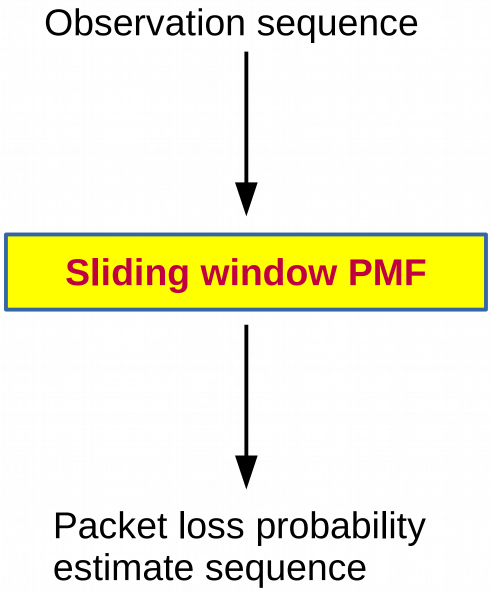
Packet Loss Probability with Stationary Time Variation

Inhomogenous Bernoulli process: Sliding Window PMF

Sliding window PMF estimation:

1) Window length:





Rectangular Sliding Window Packet Loss Probability Estimate

Packet Loss PMF Waveform & Histogram:

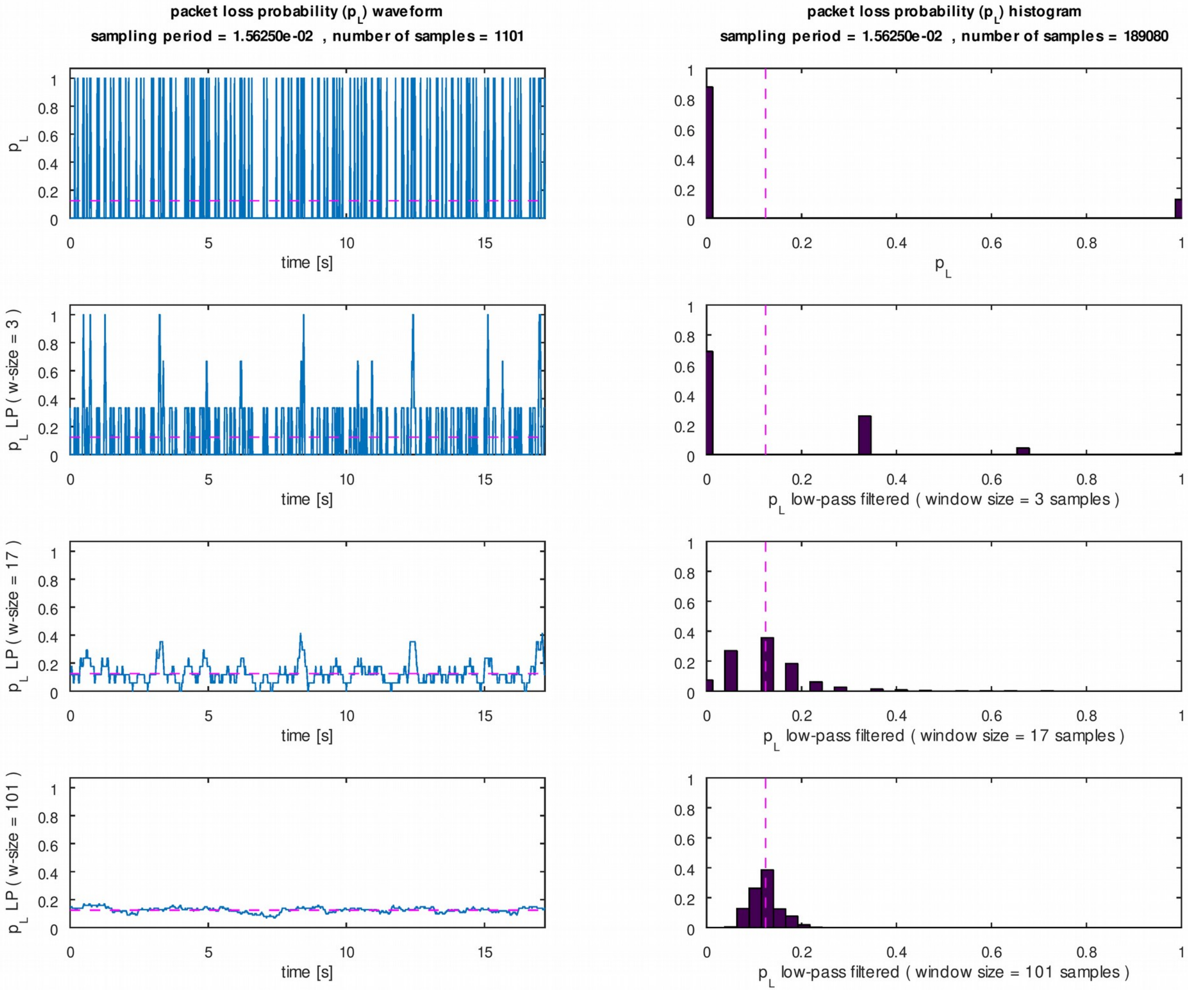
1) inhomogenous

2) low pass filtered

3) rectangular window

4) window lengths:
   [1
   3
   17
   101] samples

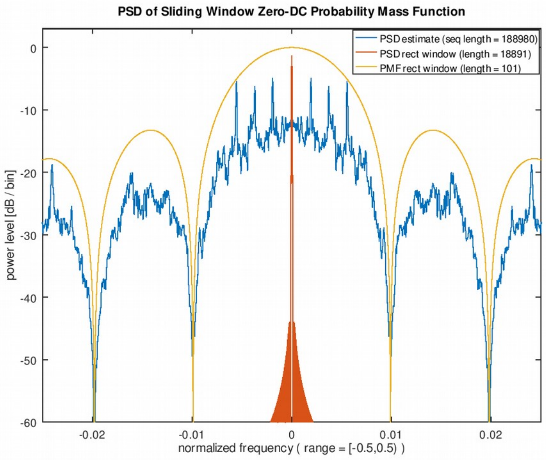
5) 64 sample/second



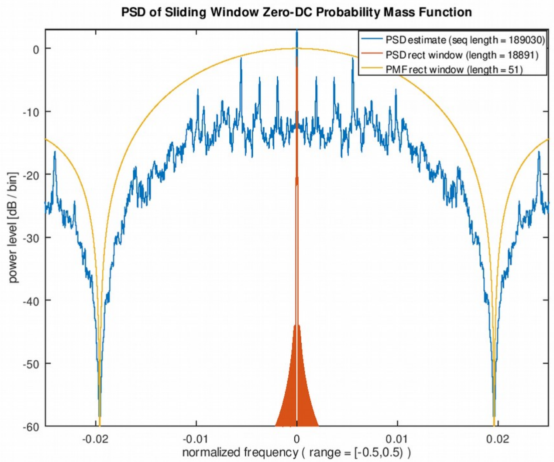
Power Spectral Density of Probability Mass Function

PSD estimation Window <u>length</u> frequency domain analysis:

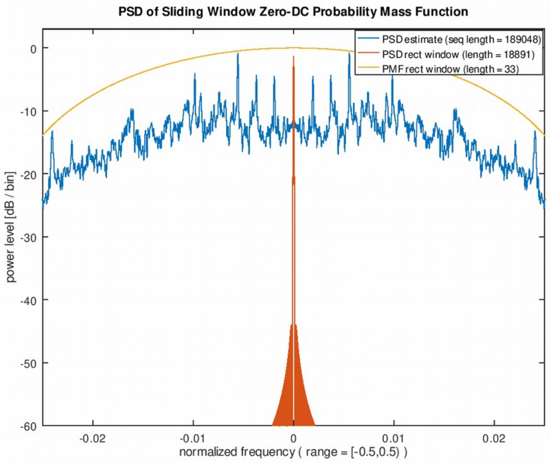
101 samples



51 samples



33 samples



4.Conclusion

Conclusion

Communication channel model must be simple & sufficiently informative for:

- NCS stability & performance analysis @ the design time
- Control system performance enhancement @ the run time through joint “communication & control” domain optimization