Non-Convex Optimization in Wireless Communications

Intended field of habilitation:

Mobile Communications

Stefan Schwarz

May 3, 2017

Convex and Non-Convex Optimization

$$\begin{aligned} &\text{minimize } f(\mathbf{x}) \\ &\text{subject to: } \mathbf{x} \in \mathcal{X} \end{aligned} \tag{1}$$

Convex optimization problems:

- Convex objective function $f(\mathbf{x}) : \mathbb{R}^n \to \mathbb{R}$, convex feasible set $\mathcal{X} \subseteq \mathbb{R}^n$
- **Efficient solvers** for many classes have become a **technology**

Non-convex optimization problems:

- Either the objective function or the feasible set is non-convex
- No effective general methods: local/global optimization
  - Implicit consideration of feasible set structure – **manifold optimization**
  - **Convex relaxation** – upper bound, approximation quality

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Scientific and Professional Career

Scientific education:

- 2004 – 2009 Dipl. Ing. Electrical Engineering, TU Wien

  Thesis: Impact of Waveguide Input Coupling on VCSELs
  Supervision: Univ. Prof. Walter Leeb, Dr. Gerhard Schmid

- 2009 – 2013 Dr. techn. Telecommunications Engineering, TU Wien

  Thesis: Limited Feedback Transceiver Design for Downlink MIMO OFDM Cellular Networks
  Supervision: Univ. Prof. Markus Rupp, *Co-examiner:* Prof. Robert W. Heath Jr., UT Austin

Project experience:

- 2008 – 2014 Project Assistant, ITC, TU Wien

  The Vienna LTE Simulators (lead developer)
  Low Latency Group Communication over LTE MBMS, General Motors (project leader), 2014

- Since 2015 University Assistant (Postdoc), ITC, TU Wien

  Full-dimension MIMO and 3D beamforming for vehicles, General Motors (project leader), 2015-2016

- Since 2016 Laboratory head of the CD-Lab for

  Dependable Wireless Connectivity for the Society in Motion

Publications: 14 journals/magazines, >30 conferences (h-index 15, ~ 970 citations)

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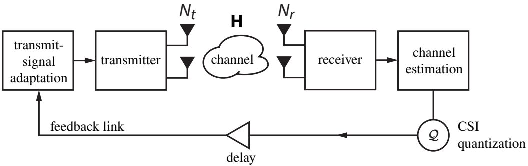
Problem Motivation[1]



- We describe the MIMO channel by a linear transformation $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$

$$\mathbf{H} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^{\mathrm{H}}, \quad \mathbf{U} \in \mathbb{C}^{N_r \times r}, \mathbf{\Sigma} \in \mathbb{R}^{r \times r}, \mathbf{V} \in \mathbb{C}^{N_t \times r} \tag{2}$$

- Channel subspace information $\mathrm{span}\,(\mathbf{V})$: **Grassmann** manifold
- Matrix of right singular vectors $\mathbf{V}$: compact **Stiefel** manifold
- Channel Gramian $\mathbf{V}\mathbf{\Sigma}^2\mathbf{V}^{\mathrm{H}}$: cone of **positive semidefinite matrices**

$\Rightarrow$ **Predictive quantization** on smooth Riemannian manifolds

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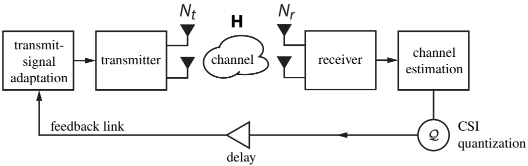

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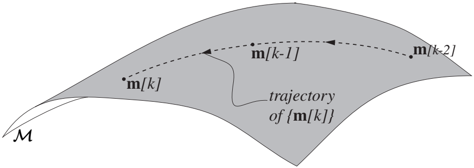
Manifold Prediction



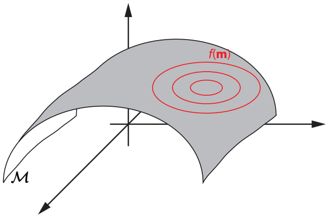
- Consider a trajectory on the manifold $\mathbf{m}(t) \in \mathcal{M}$
- We observe $\mathbf{m}$ at sampling time-instants $\mathbf{m}[k] = \mathbf{m}(kT_s)$
- Goal: predict $\mathbf{m}[k]$ from prior observations $\mathbf{m}[k-1], \mathbf{m}[k-2], \ldots$

$$\min_{\mathcal{P}} \quad \mathrm{d}^2_{\mathcal{M}}\left(\mathbf{m}[k], \hat{\mathbf{m}}[k]\right), \tag{3}$$

$$\hat{\mathbf{m}}[k] = \mathcal{P}\left(\mathbf{m}[k-1], \mathbf{m}[k-2], \ldots\right) \tag{4}$$

- Non-linear distortion metric, non-linear prediction

General Ideas of Manifold Optimization



- Goal: minimize a function $f(\mathbf{m})$ with $\mathbf{m} \in \mathcal{M}$
- Gradient optimization on embedding space + projection onto $\mathcal{M}$
- Implicit formulation on the manifold [Absil et al., 2008]
  - Charts φ map the manifold locally to the Euclidean space
  - Smooth manifold: charts φ and transition maps ψ are C^∞
  - Riemannian manifold: inner product, length, angle, distance

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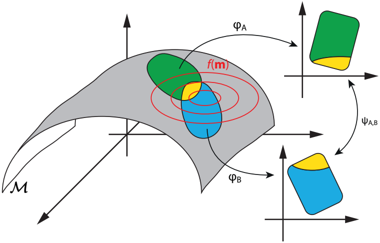

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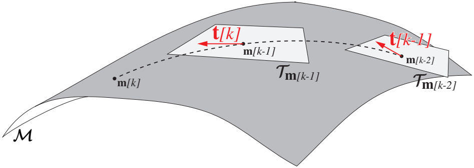
Manifold Prediction (II)



- Utilize the linear tangent vector space associated with points on the manifold[2]

$$\mathbf{t} = L\left(\mathbf{m}_1, \mathbf{m}_2\right) \in \mathcal{T}_{\mathcal{M}}(\mathbf{m}_1), \quad \mathbf{m}_2 = R\left(\mathbf{m}_1, \mathbf{t}\right) \in \mathcal{M} \tag{5}$$

- Compatible lifting/retraction pairs, e.g., exponential/logarithmic map (geodesic)
  $\Rightarrow$ bijection between curves on the manifold and tangent vectors
- Perform linear prediction in the tangent space

$$\min_{a_p} \left\| \mathbf{t}[k] - \sum_{p=1}^{N_p} a_p \bar{\mathbf{t}}[k-p] \right\|^2 \tag{6}$$

[2] *Adaptive Quantization on a Grassmann-Manifold for Limited Feedback Beamforming Systems*, S. Schwarz et al., IEEE Transactions on Signal Processing, vol 61, no 18, 2013

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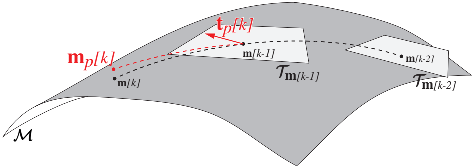

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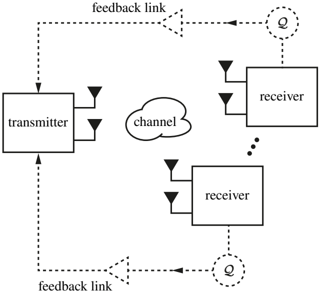
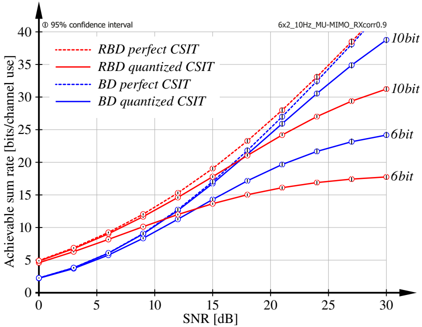
Evaluation of Multi-User MIMO Rate with Limited Feedback





- Achievable transmission rate with limited feedback[3]
- (Regularized) block-diag. [Spencer et al., 2004, Stankovic and Haardt, 2008]

[3] *Advanced Multi User MIMO Concepts*, S. Schwarz, in *The Vienna LTE-Advanced Simulators: Up and Downlink, Link and System Level Simulation*, Springer 2016

Further Extensions

- Feedback overhead reduction through excess antennas
  $\rightarrow$ subspace quantization based combining[4]

$$\min_{\mathbf{G}\in\mathbb{C}^{L\times N_r},\mathbf{Q}_i\in\mathcal{Q}} \mathrm{d}^2_{\mathcal{G}}\left(\mathbf{GH},\mathbf{Q}_i\right) \tag{7}$$

- Extension to multicarrier transmission and distributed antenna systems[5]

$$\min_{\mathbf{Q}_i\in\mathcal{Q}} \mathrm{d}^2_{\mathcal{G},\mathrm{w}}\left(\bar{\mathbf{V}},\mathbf{Q}_j,\bar{\mathbf{\Lambda}}\right), \tag{8}$$

$$\bar{\mathbf{V}}\bar{\mathbf{\Lambda}}\bar{\mathbf{V}}^{\mathrm{H}} = \bar{\mathbf{R}},\ \bar{\mathbf{R}} = \frac{1}{N}\sum_{n=1}^{N}\mathbf{V}[n]\mathbf{V}[n]^{\mathrm{H}} \tag{9}$$

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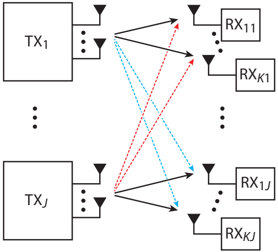
Multi-User MISO Multicast Interference Channel



- Limits of beamforming in the multi-user multicast interference channel

$$R_{kj} = \log_2\left(1 + \frac{\mathbf{h}_{kjj}^{\mathrm{H}} \mathbf{C}_j \mathbf{h}_{kjj}}{\sigma_n^2 + \sum_{\substack{\ell=1 \\ \ell \neq j}}^{J} \mathbf{h}_{kj\ell}^{\mathrm{H}} \mathbf{C}_\ell \mathbf{h}_{kj\ell}}\right), \quad \mathbf{C}_j = \mathbf{f}_j \mathbf{f}_j^{\mathrm{H}}, \tag{10}$$

$$R_j = \min_{k \in \{1,\ldots,K\}} R_{kj} \tag{11}$$

Achievable Multicast Rate Region

- Achievable rate tuples $[R_1, \ldots, R_J]$
- Weighted sum-rate optimization

$$\max_{\{\mathbf{C}_1,\ldots,\mathbf{C}_J\}} \sum_{j=1}^{J} w_j R_j, \tag{12}$$

$$\text{subject to: } \operatorname{rank}(\mathbf{C}_j) = 1$$

- Non-convex due to
  - Objective function: mutual interference coupling in R_{kj}
  - Feasible set: rank-one constraint on input covariance matrices $\mathbf{C}_j = \mathbf{f}_j \mathbf{f}_j^{\mathrm{H}}$

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Distributed Optimization[6]

- Consider transmitter j and assume all others as fixed

$$\{\mathbf{C}_1^*, \ldots, \mathbf{C}_{j-1}^*, \mathbf{C}_{j+1}^*, \ldots, \mathbf{C}_J^*\} \tag{13}$$

- Interference to users of base station j: $\Gamma_{kj\ell} = \mathrm{tr}\left(\mathbf{C}_\ell^* \mathbf{h}_{kj\ell} \mathbf{h}_{kj\ell}^{\mathrm{H}}\right)$

- Decoupled per transmitter optimization

$$\mathbf{C}_j^{\#}(\boldsymbol{\Gamma}) = \underset{\mathbf{C}_j \in \mathbb{C}^{N_t \times N_t}, \mathbf{C}_j \succeq 0}{\arg\max} \; w_j R_j, \tag{14}$$

$$\text{subject to: } \mathrm{tr}\left(\mathbf{C}_j \mathbf{h}_{i\ell j} \mathbf{h}_{i\ell j}^{\mathrm{H}}\right) \leq \Gamma_{i\ell j}, \; \forall i, \ell$$

- $\mathbf{C}_j^{\#}(\boldsymbol{\Gamma}) = \mathbf{C}_j^*$ provided $\Gamma_{i\ell j} = \mathrm{tr}\left(\mathbf{C}_j^* \mathbf{h}_{i\ell j} \mathbf{h}_{i\ell j}^{\mathrm{H}}\right)$

[6] *Transmit Optimization for the MISO Multicast Interference Channel*, S. Schwarz et al., IEEE Transactions on Communications, vol 63, no 12, 2015

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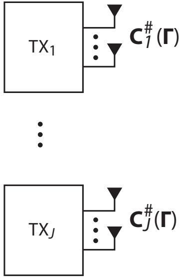
Local Distributed Optimization[6]

- Initial guess of the leakage parameters $\Gamma_{kj\ell}$
- Solve the decoupled optimization problem $\mathbf{C}_j^{\#}(\mathbf{\Gamma})$
- Determine local ascent directions $\nabla_{\mathbf{\Gamma}} L_j(\mathbf{\Gamma})$

  $\Rightarrow$ Dual-gradient approach [Zhang and Cui, 2010]
- Share local ascent directions and determine a global ascent direction (consensus)
- Iterate until vanishing improvement



6 *Transmit Optimization for the MISO Multicast Interference Channel*, S. Schwarz et al., IEEE Transactions on Communications, vol 63, no 12, 2015

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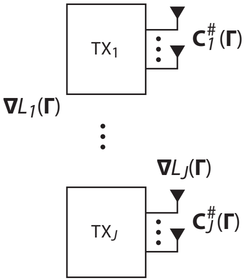

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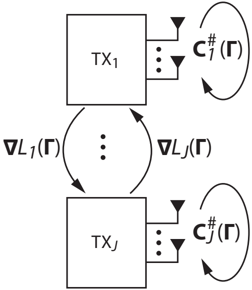

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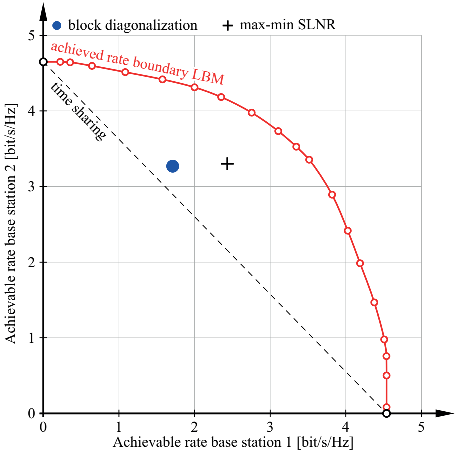
Numerical Example: Achievable Multicast Rate Region



- Two transmitters serving six users each from $N_t = 8$ antennas
- Transmission with rank-unconstrained input-covariance matrix

Probabilistic Analysis of SDR

- Beamformer randomization: $\mathbf{f}_j^{(m)} \sim \mathcal{CN}\left(\mathbf{0}, \mathbf{C}_j^*\right), \quad m = 1, \ldots, M$
- Analysis of the worst-case approximation ratio[7]

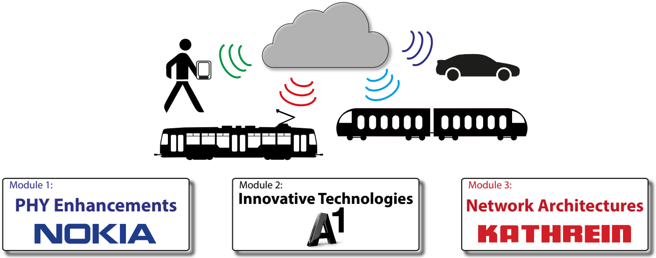
$$\frac{\mathrm{SINR}_{\mathrm{BF}}}{\mathrm{SINR}_{\mathrm{SDR}}} \geq \frac{\mu(K)}{\gamma(N)} \propto \frac{1}{K \log(N)} \tag{15}$$

- This holds with probability $\to 1$ as $M \to \infty$
- Upper bound on the rank of the optimal solution: $\mathrm{rank}(\mathbf{C}_j^*) \leq \sqrt{K + N + 1}$

[7] *Probabilistic Analysis of Semidefinite Relaxation for Leakage-Based Multicasting*, S. Schwarz, IEEE Signal Processing Letters, vol 23, no 5, 2016

Contents

CD-Lab Dependable Wireless Connectivity for the Society in Motion



- Wireless technologies to support very **large numbers of mobile users**[8]
  - Human users as well as autonomous machine-type communication
  - Efficiency, adaptability and dependability (reliability and timeliness)
  - Ranging from low-mobility (pedestrians) to very high-mobility (trains)

[8] *Society in Motion: Challenges for LTE and Beyond Mobile Communications*, S. Schwarz et al., IEEE Communications Magazine, Feature Topic: LTE Evolution, vol 54, no 5, 2016

CD-Lab Dependable Wireless Connectivity for the Society in Motion

Full-Dimension MIMO

3D beamforming

Antenna coupling

Link and system level


Multicarrier Schemes

FBMC/UFMC

Flexibility and adaptibility

Efficiency and robustness


Measurements

Extreme velocity emulation

MIMO and mmWave

Repeatability at high mobility

Heterogeneous Nets

Small cells

Distributed antennas

Stochastic geometry

System level

Vehicular Networks

V2X communications

Cellular assisted

Dependability

Stochastic geometry

mmWave Technology

Transceiver architectures

Directionality

Joint communication and Radar

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